difference between the coupling mechanisms in these two cases could be responsible for the drastically different behaviors at the cutoff, but no convincing physical interpretation can yet be offered.

ACKNOWLEDGMENT

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Computation of the Performance of the Abrupt Junction Varactor Doubler

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Abstract—**A computational procedure is given for the solution of the large signal abrupt junction varactor doubler as the input frequency is varied, given the available power of the source and the source and load impedances. The basic equations are presented in a convenient form. The steps in the procedure are then outlined, and an example is given to demonstrate their use.**

SYMBOLS

E_{ff} = efficiency of doubler
E_g = Thévenin equivalent voltage of input source
m_1 = normalized elastance coefficient at input frequency
m_2 = normalized elastance coefficient at output frequency
$\overline{P}$ = parameter defined by (8)
P_{av} = available power of source
P_{in} = input power to doubler
P_{norm} = normalization power $= (V_B+\phi)^2/R_S$
Q_{min} = charge on varactor when $S(t)=0$
q_ϕ = charge on varactor due to contact potential
R_g = real part of Thévenin equivalent source impedance
$\overline{R}_2$ = real part of $(Z_2+R_S+S_0/2j\omega)/RS$
R_S = series resistance of the varactor
S_0 = average value of $S(t)$
$S(t)$ = instantaneous value of elastance
S_{max} = value of $S(t)$ at breakdown
S_{min} = minimum value of $S(t)$
$u \triangleq (2m_2/m_1)^2$
V_B = breakdown voltage
V_0 = direct voltage across varactor
VSWR = voltage standing wave ratio
X_g = imaginary part of Thévenin equivalent source impedance
$y \triangleq m_2(\omega_c/\omega)\cos\theta$
$Z_{bb'}$ = impedance looking into terminals bb' (Fig. 1)
Z_{in} = voltage across diode at input frequency divided by current at input frequency
Z_2 = load impedance
ω = input frequency
ω_c = cutoff frequency of varactor $= S_{max}/R_S$
θ = phase angle by which input current leads output current
$\rho_{aa'}$ = reflection coefficient at terminals aa' (Fig. 4)
$\rho_{bb'}$ = reflection coefficient at terminals bb' (Fig. 4)
ϕ = contact potential
ψ_A = angle at which $S(t)$ is a maximum
ψ_B = angle at which $S(t)$ is a minimum

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INTRODUCTION

HAVING DESIGNED a doubler circuit quite often, one would like to know its efficiency and output power as the input frequency is varied about the design frequency. In this paper we therefore consider the following problem: Given an abrupt junction varactor doubler circuit whose load and source impedances have already been chosen, driven by a sinusoidal source whose frequency we are free to vary and whose available power is a known function of frequency, what is the output power and efficiency as a function of frequency? The input and output circuits may be tuned circuits and traps or more complicated networks; however, it is assumed that currents flow only at the input frequency and the second harmonic. The analysis is therefore valid only over that band of frequencies for which this assumption is approximately true.

A solution to this problem has been given[1] which involves solving a cubic equation whose coefficients are functions of the average elastance. Since the average elastance is a function of the operating point, one solves by an iteration procedure until one converges to the value of the average elastance. While such a procedure is well suited to computer solution, computation by hand is quite laborious. In this paper an iterative procedure is given which is well suited to hand computation. In the first part of the paper the basic equations are given, rewritten in the form required for their solution by this procedure. The steps in the iterative procedure are then outlined and an example given to demonstrate their use.

Basic Equations

The basic equations of the abrupt junction varactor doubler have been derived by Penfield and Rafuse.[2] The equations that we require are rewritten here. For simplicity, we have assumed $S_{\min}=0$, and hence $Q_{\min} = -q_\phi$.

$$Z_{\text{in}} = R_S + S_0/j\omega + R_S \frac{\omega_c}{\omega} m_2 e^{-j\theta} \tag{1}$$

$$Z_2 = -R_S - S_0/j2\omega + R_S \frac{\omega_c}{\omega} \frac{m_1^2}{4m_2} e^{j\theta} \tag{2}$$

$$P_{\text{in}} = 2P_{\text{norm}} \left(\frac{2\omega}{\omega_c}\right)^2 m_1^2 \left(1 + m_2 \frac{\omega_c}{\omega} \cos\theta\right) \tag{3}$$

$$E_{ff} = \frac{(\omega_c/2\omega)\cos\theta - 2m_2/m_1^2}{(\omega_c/2\omega)\cos\theta + 1/2\, m_2} \tag{4}$$

$$\frac{S(t)}{S_{\max}} = S_0/S_{\max} + 2m_1 \sin\omega t + 2m_2 \sin(2\omega t - \theta) \tag{5}$$

$$\frac{V_0 + \phi}{V_B + \phi} = \left(\frac{S_0}{S_{\max}}\right)^2 + 2m_1^2 + 2m_2^2. \tag{6}$$

Let us define three new parameters:

$$u \triangleq \left(\frac{2m_2}{m_1}\right)^2 \tag{7a}$$

$$y \triangleq m_2 \frac{\omega_c}{\omega} \cos\theta \tag{7b}$$

$$\overline{P} \triangleq \frac{2P_{\text{norm}}}{\cos^2\theta} \left(\frac{2\omega}{\omega_c}\right)^4. \tag{8}$$

In terms of these three parameters, (1) to (4) become

$$Z_{\text{in}} - R_S - S_0/j\omega = R_S y \frac{e^{-j\theta}}{\cos\theta} \tag{9}$$

$$Z_2 + R_S + S_0/2j\omega = R_S y/u \frac{e^{+j\theta}}{\cos\theta} \tag{10}$$

$$P_{\text{in}} = \overline{P} y^2 (y+1)/u \tag{11}$$

$$E_{ff} = \frac{y-u}{y+1} \tag{12}$$

and taking the real part, (10) yields

$$\overline{R}_2 \triangleq R_e\left(\frac{Z_2 + R_s + S_0/j\omega}{R_s}\right) = y/u. \tag{13}$$

Equation (11) can then be rewritten as

$$y^2 = y - \frac{P_{\text{in}}}{\overline{P}\overline{R}_2} = 0 \tag{14a}$$

$$y = 1/2\left(-1 \pm \sqrt{1 + \frac{4P_{\text{in}}}{\overline{P}\overline{R}_2}}\right). \tag{14b}$$

Equations (9) and (10) yield the relationship

$$Z_{\text{in}} = R_S + \frac{S_0}{j\omega} + u\left(Z_2 + R_S + \frac{S_0}{2j\omega}\right)^*. \tag{15}$$

This is a very important relationship since it enables us to draw a simple equivalent circuit of the input circuit of the doubler from which we will find its operating point at each frequency. The equivalent circuit is shown in Fig. 1, where E_g is the Thévenin equivalent source voltage, and R_g and X_g are the real and imaginary parts of the Thévenin equivalent source impedance. The power P_{in} is given by

$$P_{\text{in}} = P_{\text{av}}(1 - |\rho_{aa'}|^2) = P_{\text{av}}(1 - |\rho_{bb'}|^2). \tag{16}$$

However, it is more convenient to find the VSWR, using a Smith chart, and then use the relationship

$$|\rho_{bb'}| = \frac{\text{VSWR} - 1}{\text{VSWR} + 1} \tag{17}$$

to find the magnitude of the reflection coefficient.

In the calculations that follow it will be necessary to compute S_0, the average elastance for each operating point. We therefore require the relationship between S_0 and u and m_2.

For fixed bias, (6) yields

$$\frac{S_0}{S_{\max}} = \sqrt{\frac{V_0 + \phi}{V_B + \phi} - 2m_2^2\left(1 + \frac{4}{u}\right)} \tag{18}$$

while for the self-bias case, S_0 is such that $S(t)$ has a minimum value equal to zero. Equation (5) thus yields for self-bias

$$\frac{S_0}{S_{\max}} = -2m_2\left(\frac{2}{\sqrt{u}}\sin\psi_B + \sin(2\psi_B - \theta)\right) \tag{19}$$

where ψ_B is the value of ωt which minimizes $S(t)$. In Fig. 2 ψ_B is plotted for different values of u and θ.

[1] A. I. Grayzel, "The bandwidth of the abrupt junction varactor frequency doubler" (to be published).

[2] P. Penfield and R. P. Rafuse, *Varactor Applications.* Cambridge, Mass.: MIT Press, 1962, equations 832, 830, 841, 847, 850, and 849.

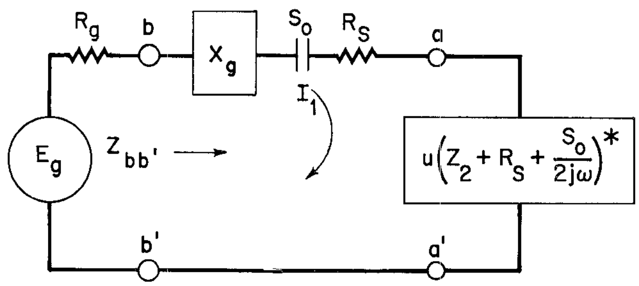

Fig. 1. Equivalent circuit of the abrupt junction frequency doubler.

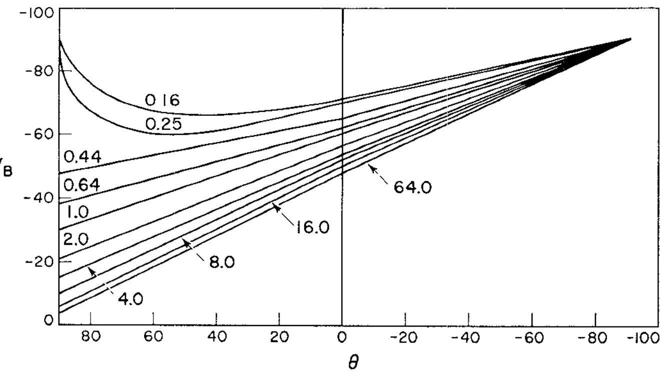

Fig. 2. ψ_B vs. θ.

We have now derived all the relationships necessary to solve for the operating point of the doubler at each frequency.

Procedure

1) Assume $S_0 = S_{\max}/2$ and calculate the real and imaginary parts of $Z_2 + R_S + S_0/2j\omega$ and the angle θ.
2) With this value of θ, calculate $\overline{P}$ using (8).
3) Assume $P_{\text{in}} = P_{\text{av}}$.
4) Find y from (14), u from (13), and m_2 from (7b).
5) Find $S_0/S_{\max}$ using (18) or (19).
6) Recompute $Z_2 + R_S + S_0/2j\omega$, θ, and $\overline{P}$ using the value of S_0 found in Step 5.
7) Find the value of $\rho_{bb'}$ (see Fig. 2) using the value of u found in Step 4 and $Z_2 + R_S + S_0/2j\omega$ found in Step 6. (A Smith chart should be used to simplify this calculation.)
8) Let $P_{\text{in}} = P_{\text{av}}(1 - |\rho_{bb'}|^2)$.
9) Repeat Steps 4 through 9 until the solution converges.

Having determined the operating point, we must now determine whether breakdown has occurred. As can be seen from (5), the conditions under which breakdown does not occur are

$$\frac{S_0}{S_{\max}} + 2m_2\left[\frac{2}{\sqrt{u}}\sin\psi_A + \sin(2\psi_A - \theta)\right] \leq 1 \quad (20)$$

$$\frac{S_0}{S_{\max}} + 2m_2\left[\frac{2}{\sqrt{u}}\sin\psi_B + \sin(2\psi_B - \theta)\right] \geq 0 \quad (21)$$

where ψ_A is the value of $\omega(t)$ which maximizes $S(t)$ and ψ_B the value which minimizes it. It can be shown[3] that:

$$\psi_A(\theta) = -\psi_B(-\theta) \quad (22)$$

and hence ψ_A and ψ_B can be obtained from Fig. 2. Substitution of ψ_A, ψ_B, u, m_2, and θ in (20) and (21) will then determine whether breakdown has occurred.

Having determined u and y, the efficiency can be determined from (4). The output power is then given by

$$P_{\text{out}} = E_{ff}P_{\text{in}} = P_{\text{av}}(1 - |\rho_{bb'}|^2)E_{ff}. \quad (23)$$

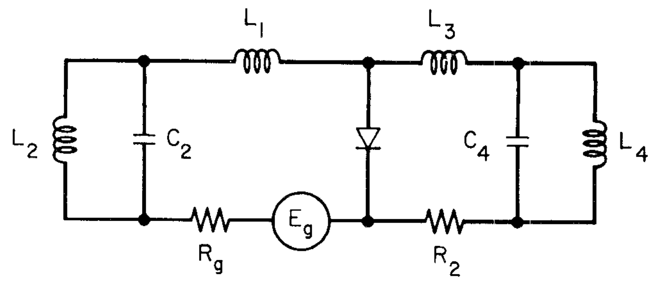

Fig. 3. Doubler circuit, Example 1.

TABLE I

R_g	L_1	L_2	C_2	L_3	L_4	C_4	R_2
39.8	0.375	0.100	6.35	0.339	0.635	4.00	18.8

Example 1

Let us consider the doubler circuit of Fig. 3. The varactor has the following parameters:

$\omega_c = 400$ Gc/s, $C_{\min} = 2.5$ pF, $R_S = 1$, $V_B + \phi = 100$ volts.

The element values are given in Table I. The available power of the source is 55.5 mW and the varactor is fixed bias such that $V_0 + \phi = 27$ volts. The doubler was designed such that at an input frequency of 100 Mc/s the input and output circuits are resonant and the source is matched. This can be verified in the following manner: At 100 Mc/s

$$\frac{S_{\max}}{\omega} = 637$$

and if we assume that $S_0 = S_{\max}/2$

$$\overline{R}_2 = 18.5 \qquad X_2 = 0 \qquad \cos\theta = 0$$

$$R_g = 39.8 \qquad X_g = 318.5.$$

Since $X_g = S_0/\omega$ and $\cos\theta = 0$, we see that the input and output circuits are both resonant. Equation (8) yields $\overline{P} = 1.948 \times 10^{-6}$ and if we assume $P_{\text{in}} = P_{\text{av}}$ then $P_{\text{in}}/\overline{P} = 28.5 \times 10^3$, and (14b) then yields $y = 38.8$. Taking the real part of (9) yields

$$\text{Re}(Z_{\text{in}}) = 1 + y = 1 + 38.8 = 39.8.$$

[3] A. I. Grayzel, "The bandwidth of the abrupt junction varactor frequency doubler," Ph.D. dissertation, Massachusetts Institute of Technology, Cambridge, Mass., 1963, pp. 27–28.

We thus see that the source is matched.

Let us now calculate the performance of the doubler at 95 Mc/s. At 95 Mc/s

$$\frac{S_{\max}}{\omega} = 670.5.$$

Step 1: Assume $S_0 = S_{\max}/2$, then

$$\overline{R}_2 = 18.5 \quad X_2 = -52.8. \quad \cos\theta = 0.331$$
$$R_g = 39.8 \quad X_g = 300.75.$$

Step 2:

$$\overline{P} = \frac{2 \times 10^4}{(0.331)^2}[2 \times 1.49 \times 10^{-3}]^4 = 18.25 \times 10^6.$$

Step 3: Let $P_{in} = P_{av} = 55.5$ mW, then

$$\frac{P_{in}}{\overline{P}} = 3.04 \times 10^3.$$

Step 4:

$$y = -1 + \frac{\sqrt{1 + (4)(3.04)10^{-3}}}{18.5} = 12.3$$

$$u = \frac{12.3}{18.5} = 0.665$$

$$m_2 = \frac{12.3 \times 1.49 \times 10^{-3}}{0.331} = 0.055.$$

Step 5:

$$\frac{S_0}{S_{\max}} = \sqrt{0.27 - (2)(0.055)^2\left(1 + \frac{4}{0.665}\right)} = 0.476.$$

Step 6: Using this new value of $S_0/S_{\max}$ we now recompute $Z_g + R_S + S_0/2j\omega$. The real part does not change.

S_0/ω changes by $(0.5 - 0.476)670.5 = 15$

and

$S_0/2\omega$ changes by 7.5.

Hence,

$$\frac{S_0}{\omega} = 320.25$$

$$\overline{R}_2 = 18.5 \quad X_2 = -45.3$$
$$\cos\theta = 0.378 \quad \overline{P} = 14 \times 10^{-6}.$$

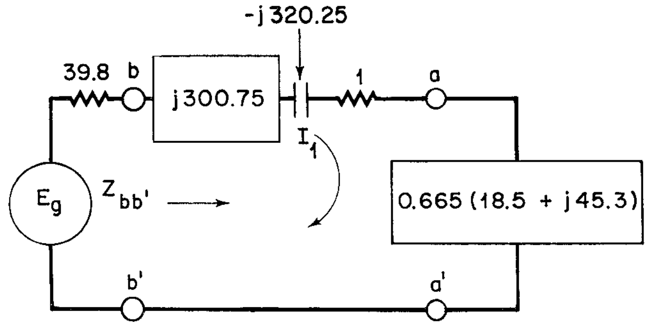

Fig. 4. Equivalent circuit, Example 1 (first iteration).

TABLE II

Iteration	1	2	3	4
$\overline{R}_2$	18.5	18.5	18.5	18.5
X_2	−52.8	−45.3	−49.4	−48.1
$\cos\theta$	0.331	0.378	0.351	0.360
$\|\rho\|$	0	0.524	0.535	0.565
$\overline{P}$	18.25×10^{-6}	14×10^{-6}	16.2×10^{-6}	15.4×10^{-6}
$P_{in}/\overline{P}$	3.04×10^3	2.87×10^3	2.44×10^3	2.5×10^3
y	12.3	12	11	11.15
u	0.665	0.649	0.595	0.60
m_2	0.055	0.047	0.047	0.046
$S_0/S_{\max}$	0.476	0.490	0.486	0.488

Step 7: The equivalent circuit of Fig. 1 takes on the values shown in Fig. 4, and $\rho_{bb'} = 13.2 + j10.4$. Normalizing to 39.8 Ω and plotting on a Smith chart, one finds a VSWR of 3.2. Then $|\rho|$ equals

$$\frac{3.2 - 1}{3.2 + 1} = 0.524.$$

Step 8: Letting $P_{in} = P_{av}(1 - |\rho|^2)$, then $P_{in}/\overline{P} = 2.87 \times 10^{-3}$.

We now return to Step 4 and repeat through Step 8. The calculated values are shown in Table II for four iterations. The operating point converges to

$$u = 0.60, \quad m_2 = 0.046, \quad S_0/S_{\max} = 0.488.$$

Substituting these values into (20) and (21), we find

$$0.04 < \frac{S(t)}{S_{\max}} < 0.74$$

and hence the diode is not in the breakdown region. The efficiency is found from (12) to equal 0.868, and using (23), $P_{out}/P_{av} = 0.602$.